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EXACT PROBABILITIES FOR THE WILCOXON

SIGNED RANK TEST

by

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Exact Probabilities For The Wilcoxon
Signed Rank Test

by

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A compact set of tables and a simple algorithm for using these tables to compute exact null probabilities of the Wilcoxon signed rank test is presented. The tables supplied permit the computation of any null probability when the sample size is less than or equal to 36. Null probabilities for values of the test statistic ≤ 299 or greater than $\frac{n(n+1)}{2} - 299$, where n is the sample size, can be obtained from these

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tables for any sample size.

EXACT PROBABILITIES FOR THE WILCOXON SIGNED RANK TEST

The Wilcoxon Signed Rank Test

The Wilcoxon Signed Rank Test is a well known distribution-free test of location for the population median, proposed by the late Professor Frank Wilcoxon (1). For completeness, a brief description of the test is presented below.

Suppose (X_1, \dots, X_n) is a random sample drawn from a population governed by a symmetric continuous probability law, and we wish to test the null hypothesis, H_0 , that the center of symmetry of the population is some value μ_0 . Then the Wilcoxon test statistic W_+ may be computed as follows:

Compute (Z_1, \dots, Z_n) where $Z_i = (X_i - \mu_0)$, and rank the Z_i by absolute magnitude. W_+ is the sum of the ranks of all the positive Z_i in the sample.

The results in this paper provide a relatively compact set of tables and a simple method for using these tables to compute the exact null probabilities for W_+ , i.e. the $P(W_+ = w | H_0)$ or $P(W_+ \leq w | H_0)$.

Exact probabilities for the null distribution of W_+ are already available in table form for sample sizes less than or equal to 20, Owen(7). For sample sizes larger than 20, two approximations are known to the author, the normal approximation and an Edgeworth approximation developed by Fellingham and Stoker (5). Fellingham and Stoker (5) also report the existence of an unpublished paper by Ury and Chacko (6) which gives a method for computing the exact null probabilities of W_+ from the Fix and Hougues (2) tables for the two sample Wilcoxon. A comprehensive table of

significance points has been published by McCornach (3), who also provides an evaluation of the normal approximation, in which he shows the normal approximation to be in general too small at the .05 level and too large at the .005 level of significance. Though for sample sizes greater than thirty-five the relative error at these significance levels is less than ten per cent. The tables and method presented here permit the easy calculation of probabilities over the entire null distribution of W_+ for sample sizes less than or equal to 35 as well as probabilities for values of W_+ less than 300 or greater than $\frac{n(n+1)}{2} - 253$, when n , the sample size is larger than 35.

The Null Distribution of W_+

As a result of the procedure described in the last paragraph, we arrive at some value w for W_+ (where $0 \leq w \leq \frac{n(n+1)}{2}$). The number of possible ways of obtaining the value w is the number of partitions of w into distinct positive integral parts which do not exceed n , i.e., the number of ways of picking subsets of the integers 1, 2, ..., n , so that the sum of any subset is equal to w . If we call the number of such partitions $a(w, n)$, then under the null hypothesis that the center of symmetry is μ_0 , the

$$P \{ W_+ = w \} = \frac{a(w, n)}{2^n} \quad (1)$$

$$= \frac{\text{number of partitions of } w \text{ into distinct positive integral parts} \leq n}{2^n}$$

We now present a method for calculating the value of $a(w, n)$, patterned after the work of Fix and Hodges on the Two Sample Wilcoxon (2).

An Algorithm for Computing $a(w, n)$

It is a natural extension of the definition of $a(w, n)$ to define $a(w, \infty) = a(w)$, as the number of partitions of w into distinct positive integral parts, and since no partition of w may contain parts greater than w , it is clear that for $w \leq n$, $a(w) = a(w, n)$. Extensive tables for $a(w)$ have been compiled (4).

For $w > n$,

$$a(w, n) = a(w) - (\text{The number of partitions* of } w \text{ whose largest part is greater than } n) \quad (2)$$

*Throughout the rest of this discussion, unless otherwise stated, the term partition will be used to mean partitions into positive integral parts.

Lemma

(The number of partitions of w whose largest part is greater than n) =

$$\sum_{t=n}^{\infty} a(w-(t+1), t).$$

Proof

$a(w-(t+1), t)$ = number of partitions of $(w-(t+1))$
into parts not exceeding t

but $(w-(t+1)) + (t+1) = w$, hence

$a(w-(t+1), t)$ = number of partitions of w whose
largest part is $t+1$

and so

$$a(w-(n+1), n) + a(w-(n+2), n+1) +$$

$$a(w-(n+3), n+2) + \dots$$

$$= \sum_{t=n}^{\infty} a(w-(t+1), t)$$

is the number of partitions of w whose largest part
is greater than n

Applying this lemma to equation (2) produces

$$a(w, n) = a(w) - \sum_{t_1=n}^{\infty} a(w-(t_1+1), t_1) \dots \dots \dots (3)$$

and equation (3) in its turn implies that

$$a(w-(t_1+1), t_1) = a(w-(t_1+1)) - \sum_{t_2=t_1}^{\infty} a(w-(t_1+1)-(t_2+1), t_2) \dots \dots (4)$$

Substituting (4) back into (3) permits the expansion

$$a(w, n) = a(w) - \sum_{t_1=n}^{\infty} a(w-(t_1+1)) + \sum_{t_1=n}^{\infty} \sum_{t_2=t_1}^{\infty} a(w-(t_1+1)-(t_2+1), t_2) \dots (5)$$

Continued use of equation (4) to expand equation (5)

further produces the result

$$\begin{aligned} a(w, n) &= a(w) - \sum_{t_1=n}^{\infty} a(w-(t_1+1)) + \sum_{t_1=n}^{\infty} \sum_{t_2=t_1}^{\infty} a(w-(t_1+1)-(t_2+1)) \\ &\quad - \sum_{t_1=n}^{\infty} \sum_{t_2=t_1}^{\infty} \sum_{t_3=t_2}^{\infty} a(w-(t_1+1)-(t_2+1)-(t_3+1)) + \dots \\ &\quad + (-1)^k \sum_{t_1=n}^{\infty} \sum_{t_2=t_1}^{\infty} \dots \sum_{t_k=t_{k-1}}^{\infty} a(w-(t_1+1) - \dots - (t_k+1)) + \dots \dots \dots (6) \end{aligned}$$

Applying the transformation,

$$\begin{aligned}s_1 &= t_1 - n \\ s_2 &= t_2 - t_1 \\ s_3 &= t_3 - t_2 \\ &\vdots \\ s_k &= t_k - t_{k-1},\end{aligned}$$

to equation (6) we obtain

$$\begin{aligned}a(w, n) &= a(w) - s_1 \frac{\partial}{\partial 0} a(w - n - 1 - s_1) + s_1 \frac{\partial}{\partial 0} s_2 \frac{\partial}{\partial 0} a(w - 2n - 2 - 2s_1 - s_2) \\ &\quad - s_1 \frac{\partial}{\partial 0} s_2 \frac{\partial}{\partial 0} s_3 \frac{\partial}{\partial 0} a(w - 3n - 3 - 3s_1 - 2s_2 - s_3) + \\ &\quad \dots + (-1)^k s_1 \frac{\partial}{\partial 0} \dots s_k \frac{\partial}{\partial 0} a(w - kn - k - ks_1 - (k-1)s_2 - \dots - s_k) + \dots\end{aligned}\quad (7)$$

If we now define the quantities $A_1(u)$ as**

$$A_1(u) = \frac{\partial}{\partial 0} a(u - t)$$

$$A_2(u) = \frac{\partial}{\partial 0} A_1(u - 2t)$$

$$A_3(u) = \frac{\partial}{\partial 0} A_2(u - 3t)$$

and in general

$$A_k(u) = \frac{\partial}{\partial 0} A_{k-1}(u - kt)$$

** Note that for $u < 0$ $a(u) = 0$

then from equation (7), we can write

$$\begin{aligned}
 a(w,n) &= a(w) - A_1(w-n-1) + s_1 \sum_{i=0}^w A_1(w-2n-2-2s_1) \\
 &\quad - s_1 \sum_{i=0}^w s_2 \sum_{j=0}^w A_1(w-3n-3-3s_1-2s_2) + \dots \\
 &= a(w) - A_1(w-n-1) + A_2(w-2n-2) - s_1 \sum_{i=0}^w A_2(w-3n-3-3s_1) \\
 &= a(w) - A_1(w-n-1) + A_2(w-2n-2) - A_3(w-3n-3) + \\
 &\quad \dots + (-1)^k A_k(w-kn-k) + \dots
 \end{aligned} \tag{8}$$

Tables for $A_{k-1}(kj)$ are easily computed from the tables of $a(w)$ in (4), and tables for $w < 300$ and $k < 9$ are presented in this table

The Use of Equation (8) to Compute Probabilities for W_+ - Some Examples

For the sample size $n = 4$ and $W_+ = 9$, the

$$\begin{aligned}
 P(W_+ = 9 | n=4) &= 2^{-4} a(9,4) \\
 &= 2^{-4} \{a(9) - A_1(4) + A_2(1)\} \\
 &= 2^{-4} (8-7) \\
 &= 2^{-4}
 \end{aligned}$$

When, as in the case above, the observed value of W_+ is greater than $\frac{n(n+1)}{4}$ the relationship $P(W_+ = w | n) = P(W_+ = \frac{n(n+1)}{2} - w | n)$ (9)

can be used to reduce the number of computational steps required. If the observed value of W_+ is greater than the greatest table value, in this case 300, then the relationship given in (9) must be used. e.g.

$$\begin{aligned}
 P(W_+ = 400 | n=30) &= P(W_+ = \frac{30(31)}{2} - 400 | n=30) \\
 &= P(W_+ = 65 | n=30) \\
 &= 2^{-30} (a(65) - A_1(65-31) + A_2(65-62)) \\
 &= 2^{-30} (a(65) - A_1(34) + A_2(3)) \\
 &= 2^{-30} (18200 - 3725 + 7) \\
 &= 2^{-30} (14,482) \\
 &= 0.00001
 \end{aligned}$$

CUMULATIVE PROBABILITIES

A similar expression for the cumulative probabilities for W_+ can be obtained from equation (8), i.e.

$$\begin{aligned}
 P(W_+ \leq w, n) &= \frac{\sum_{u=0}^w a(u, n)}{2^n} \\
 &= 2^{-n} (A_1(w) - \sum_{u=0}^w A_1(u-n-1) + \sum_{u=0}^w A_2(u-2n-2) \\
 &\quad - \sum_{u=0}^w A_3(u-3n-3) + \dots + (-1)^k \sum_{u=0}^w A_k(u-kn-k) \\
 &\quad \dots) \\
 &= 2^{-n} (A_1(w) - SA_1(w) + SA_2(w) - SA_3(w) \\
 &\quad + \dots + (-1)^k SA_k(w) + \dots), \tag{10}
 \end{aligned}$$

$$\text{where } SA_i = \sum_{u=0}^w A_i(u-in-i)$$

Tables for A_i and the SA_i are included in this paper for $i \leq 9$. These tables can be used to obtain complete coverage of the null distribution for sample sizes less than or equal to 35 as well as probabilities for values of W_+ less than 300 or greater than $\frac{n(n+1)}{2} - 200$, when n the sample size is greater than 35.

An Example of the Use of Equation (8)

For a sample size of 30 and an observed W_+ of 400, the symmetry of the distribution of W_+ implies

$$\begin{aligned}
 P(W_+ \leq 400, n=30) &= 1 - P(W_+ \leq 65, n=30) \\
 &= 1 - 2^{-30} (A_1(65) - SA_1(34) + SA_2(3)) \\
 &= 1 - 2^{-30} (176945 - 24883 + 14) \\
 &= 1 - 2^{-30} (152076) \\
 &= \frac{1073741624 - 152076}{1073741624} \\
 &= 0.99986
 \end{aligned}$$

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Tables of $a(w)$ for Computing Probabilities
of the Wilcoxon Signed Rank Statistic

w	0	1	2	3	4
10	1	1	1	2	2
20	10	12	15	18	22
30	64	76	89	104	122
40	296	340	390	448	512
50	1113	1260	1426	1610	1816
60	3658	4097	4582	5120	5718
70	10880	12076	13394	14848	16444
80	29927	32992	36352	40026	44046
90	77312	84756	92864	101698	111322
100	189586	206848	225585	245920	267968
110	444793	483330	525016	571078	618784
120	1004544	1087744	1177438	1271118	1378304
130	2194432	2368800	2556284	2757826	2974400
140	4654670	5010688	5392550	5802008	6240974
150	9617150	10327156	11086968	11899934	12769602
160	19406016	20792120	22272512	23853318	25540982
170	38328320	40982540	43812110	46828032	50042056
180	74236384	79229676	84543782	90198446	96214550
190	141231780	150473568	160293888	170727424	181810744
200	264288462	281138048	299016608	317984256	338104630
210	487067746	517361670	549462336	583473184	619503296
220	884987529	938748852	995645336	1055852590	1119555488
230	1586861606	1681116852	1780751883	1886061684	1997357056
240	2810394454	2973772212	3146284870	3328423936	3520706304
250	4919887992	5200062976	5495597248	5807301632	6136027874
260	8519280128	8994926602	9496158208	10024300890	10580747264
270	14600965705	15400801856	16242882560	17129359744	18062490974
280	24782061070	26114971540	27517053882	28991772486	30542758738
290	41676826624	43879178240	46193897032	48626519094	51182844672
300	69479740554	73089209120	76879839744	80860419136	85040145750

TO READ TABLES:

Each table row has entries for ten different values of w , i.e., the first row has values of $a(w)$ (or $A_1(w)$ or $SA_1(w)$ as appropriate) for $w=0, 1, \dots, 9$, the second has $a(w)$ for $w = 10, \dots, 19$, etc. To find the value $a(w)$, $A_1(w)$, or $SA_1(w)$ for some w , use the appropriate row and column, e.g., for $w = 153$ $a(w)$ is the value given in the fourth column of the row which is numbered 150, i.e. $a(153) = 23853318$.

Tables of a (w) for Computing Probabilities (con't)

W	5	6	7	8	9
	3	4	5	6	8
10	27	32	38	46	54
20	147	165	192	222	256
30	585	668	760	864	982
40	2048	2304	2590	2910	3264
50	6378	7108	7917	8808	9792
60	18200	20132	22250	24576	27130
70	48446	53250	58499	64234	70488
80	121792	133184	145578	159046	173682
90	291874	317788	345856	376256	409174
100	671418	728260	789640	855906	927406
110	1490528	1611388	1741521	1881578	2032290
120	3207086	3457027	3725410	4013544	4322816
130	6711480	7215644	7755776	8334326	8953856
140	13699699	14694244	15757502	16893952	18108418
150	27342421	29264960	31316314	33504746	35839008
160	53466624	57114844	61000704	65139008	69545358
170	102614114	109420549	116658616	124354422	132535702
180	193582642	206084096	219358315	233451098	248410816
190	359444904	382075868	406072422	431513602	458482688
200	657667584	698087424	740890786	786212446	834194700
210	1186949056	1258238720	1333640710	1413383026	1497705768
220	2114965120	2239229960	2370513986	2509198528	2655684608
230	3723675326	3937902688	4163989458	4402567324	4654300706
240	6482671322	6848172604	7233519619	7639750522	8067955712
250	11166959338	11784471548	12434895064	13119920928	13841323582
260	19044644146	20078303620	21166075136	22310691192	23515017984
270	32173819904	33888946600	35592320960	37588326642	39581557440
280	53868949522	56691197084	59656252987	62771098024	66043042088
290	89428647940	94036004868	98872765938	103949971456	109279176298

TABLES OF $A_1(W)$ FOR COMPUTING PROBABILITIES

W	0	1	2	3	4
	1	2	3	5	7
10	43	55	70	88	110
20	371	447	536	640	762
30	2035	2375	2765	3213	3725
40	8697	9957	11383	12993	14809
50	31583	35680	40262	45382	51100
60	101983	114059	127453	142301	158745
70	300960	333952	370304	410330	454376
80	826605	911361	1004225	1105923	1217245
90	2140113	2346961	2572546	2818466	3086434
100	5272175	5755505	6280521	6850599	7469383
110	12446557	13534301	14711739	15985857	17364161
120	28315898	30684698	33240982	35998808	38973208
130	62353761	67364449	72756999	78559007	84799981
140	133388213	143715369	154802337	166702271	179471873
150	278031704	298823824	321096336	344949654	370490636
160	566086405	607068945	650881055	697709087	747751143
170	1128254065	1207483741	1292027523	1382225969	1478440519
180	2205255702	2355729270	2516023158	2686750582	2868561326
190	4233736755	4514874803	4813891411	5131875667	5469980297
200	7994637527	8511999197	9061461533	9644934717	10264438013
210	14866478482	15805227334	16800872670	17856725260	18976280748
220	27253059634	28934176486	30714928369	32600990053	34598347109
230	49298333765	52272105977	55418390847	58746814783	62267521087
240	88069844581	93269907557	98765504805	104572806437	110708834311
250	155500184218	164495110820	173991269028	184015569918	194596317182
260	271544853347	286945655203	303188537763	320317897507	338380388481
270	469277181629	495392153169	522909207051	551900979537	582443738275
280	803045536445	846924714685	893118611717	941745130811	992927975483
290	1361438255742	1434527464862	1511407304606	1592267723742	1677307869492

TABLES OF $A_1(W)$ FOR COMPUTING PROBABILITIES

W	5	6	7	8	9
	10	14	19	25	33
10	137	169	207	253	307
20	904	1069	1261	1483	1739
30	4310	4978	5738	6602	7584
40	16857	19161	21751	24661	27925
50	57478	64586	72503	81311	91103
60	176945	197077	219327	243903	271033
70	502822	556072	614571	678805	749293
80	1339037	1472721	1617799	1776845	1950527
90	3378308	3696096	4041952	4418208	4827382
100	8140801	8869061	9658701	10514607	11442013
110	18854689	20466077	22207598	24089176	26121466
120	42180294	45637321	49362731	53376275	57699091
130	91511461	98727105	106482881	114817207	123771063
140	193171572	207865816	223623318	240517270	258625688
150	397833057	427098017	458414331	491919077	527758085
160	801217767	858332611	919333315	984472323	1054017681
170	1581054633	1690475182	1807133798	1931488220	2064023922
180	3062143968	3268228064	3487586379	3721037477	3969448293
190	5829425201	6211501069	6617573491	7049087093	7507569781
200	10922105597	11620193021	12361083807	13147296253	13981490953
210	20163229804	21421468524	22755109234	24168492260	25666198028
220	36713312229	38952542189	41323056175	43832254703	46487939311
230	65991196413	69929099101	74093088559	78495655883	83149956589
240	117191505633	124039678237	131273197856	138912948378	146980904090
250	205763276520	217547748068	229982643132	243102564060	256943887642
260	357425032627	377503336247	398669411383	420980102575	444495120559
270	614617558179	648506504779	684198825739	721787152381	761368709821
280	1046796925005	1103488122089	1163144375076	1225915473100	1291958515188
290	1766736517432	1860772522300	1959645288238	2063595259694	2172874435992

TABLES OF $A_2(W)$ FOR COMPUTING PROBABILITIES

W	0	1	2	3	4
10		93	124	163	212
20		1066	1310	1602	1950
30		6951	8229	9716	11442
40		33718	39031	45101	52024
50		135315	154237	175577	199619
60		474557	534762	602010	677063
70		1502695	1678320	1872999	2088650
80		4388857	4866697	5393082	5972620
90		11999506	13226944	14572052	16045410
100		31044965	34048557	37325486	40899156
110		76625094	83674972	91336833	99660829
120		181572145	197529280	214813127	233528088
130		415153692	450134653	487910691	528693660
140		919643197	994174434	1074445534	1160876705
150		1980332197	2135121107	2301428533	2480070761
160		4157022668	4471145179	4807903723	5168854266
170		8526713865	9150906770	9818741388	10533132739
180		17124401011	18341074362	19640424169	21027824944
190		33731987791	36061878387	38545879202	41193754054
200		65271085188	69660321724	74332546721	79305256441
210		124230952490	132375164132	141031825160	150231887392
220		232851126326	247750602944	263566054695	280351592997
230		430247532461	457148006689	485665923308	515894821472
240		784428043960	832398970590	883193548765	936971777027
250		1412355193909	1496912495426	1586346462937	1680928065344
260		2513137945594	2660563527841	2816326483357	2980881425348
270					
280					
290					
300					
310					
320					
330					
340					
350					
360					
370					
380					
390					
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TABLES OF $A_3(W)$ FOR COMPUTING PROBABILITIES

W	0	1	2	3	4
10		142	195	265	354
20		2090	2619	3261	4040
30		16049	19260	23043	27491
40		87909	102847	120085	139933
50		388704	446935	513138	588323
60		1477769	1677571	1902299	2154832
70		5014356	5636362	6330031	7103006
80		15558637	17350452	19334871	21531257
90		44889741	49732865	55065840	60935151
100		121901986	134310905	147908271	162801142
110		314429526	344801737	377942576	414090355
120		775797992	847243621	924915135	1009326080
130		1841254761	2003558101	2179430631	2369948421
140		4222599972	4580065170	4966290797	5383476677
150		9391986369	10157760674	10982985716	11872057130
160		20323020660	21922970976	23643054273	25491874926
170		42894745055	46162492876	49668026820	53427877794
180		88505396964	95042433863	102041304144	109533221908
190		178860962440	191691402479	205403318253	220054716494
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TABLES OF $A_2(W)$ FOR COMPUTING PROBABILITIES

W	4	6	7	8	9
	17	25	36	50	69
10	349	442	556	695	863
20	2854	3433	4115	4916	5854
30	15752	18419	21490	25021	29074
40	68881	79071	90632	103732	118557
50	257097	291263	329600	372574	420703
60	854008	957832	1073335	1201735	1344368
70	2591472	2883447	3206043	3562252	3955336
80	7311657	8082548	8929456	9859393	10879983
90	19423718	21354582	23465670	25777790	28293052
100	49039957	53663930	58698658	64178537	70140671
110	118515518	129167071	140723116	153256247	166844582
120	275708382	299423656	325071113	352799931	382770204
130	620205121	671437777	726688002	786254984	850459065
140	1354048277	1461783223	1577671595	1702300493	1836297283
150	2877903818	3099017186	3336318149	3590936263	3864076234
160	5970072033	6413987477	6889405348	7398459800	7943423029
170	12114187372	12987657089	13921321170	14919145309	15985345092
180	24089968912	25777213559	27577555291	29498251036	31547003584
190	47023179255	50227360568	53640752746	57276447661	61148322527
200	90227362038	96217177755	102588445845	109364474008	116569936798
210	170395119196	181429574432	193150228430	205598066692	218816426458
220	317064905226	337116943993	358387961401	380949198696	404875900712
230	581886017885	617862543496	655979106444	696358199379	739129063033
240	1054163282660	1117942061313	1185436480516	1256855009691	1332417384606
250	1886691341864	1998490528187	2116673984996	2241593092247	2373617872638
260	3338306457975	3532210208085	3736975869358	3953190310660	4181470987917

TABLES OF $A_3(W)$ FOR COMPUTING PROBABILITIES

W	5	6	7	8	9
	21	33	49	71	102
10	614	796	1024	1309	1659
20	6115	7473	9098	11031	13327
30	38795	45910	54191	63816	74984
40	188966	219004	253389	292698	337561
50	770235	879586	1003212	1142809	1300289
60	2756307	3112664	3511661	3958042	4457032
70	8921503	9986453	11169780	12483755	13941789
80	26646528	29613805	32890235	36505921	40493788
90	74489558	82289733	90857021	100262348	110582785
100	196948228	216465072	237804432	261126765	286605743
110	496458094	543257426	594225847	649714341	710102008
120	1200623517	1308749736	1426101069	1553423448	1691519940
130	2799635752	3041386198	3302956775	3585890736	3891845263
140	6320339074	6845259900	7411654172	8027639567	8681557183
150	13860889534	14971074316	16165997992	17451825797	18835150550
160	29613126306	31905862403	34368031190	37011586106	39849285432
170	61782214192	66415534883	71380995953	76701359501	82400879975
180	126131273056	135310435467	145128974649	155629524092	166857439051
190	252426497508	270282077062	289348014724	309702945169	331430399589
200	495990308348	530141534108	566548697472	605354782356	646711470906
210	958138415262	1022441113786	1090888280826	1163736481954	1241257540244
220	1821888500165	1941207944142	2068039448103	2202837698861	2346083844854

TABLES OF $A_u(W)$ FOR COMPUTING PROBABILITIES

W	0	1	2	3	4
	1	2	4	8	14
10	179	252	350	479	647
20	3236	4122	5217	6565	8219
30	28739	34923	42293	51055	61440
40	176112	208093	245341	288635	338869
50	853049	988959	1144705	1322943	1526661
60	3502060	4003726	4571769	5214315	5940386
70	12698789	14362338	16228459	18320071	20662526
80	41773549	46840303	52481152	58756887	65734328
90	126984698	141379987	157306089	174915770	194376049
100	361497808	400083696	442546668	489253472	540603582
110	973441266	1071859641	1179672923	1297728022	1426943997
120	2498728341	2739197490	3001573473	3287745823	3599758297
130	6151577970	6717404993	7332612376	8001289368	8727846743
140	14596598546	15884311313	17280028276	18792362685	20430581123
150	33517274545	36361777531	39436206406	42758264700	46346954388
160	74730301382	80847233668	87441834458	94549692541	102208927224
170	162242441916	175080216607	188888540150	203737523200	219702116699
180	343809471997	370160953016	398444780344	428796532682	461360891355
190	712616178251	765616909810	822393733700	883204381617	948323440229

TABLES OF $A_s(W)$ FOR COMPUTING PROBABILITIES

W	0	1	2	3	4
	1	2	3	4	
10	203	291	411	572	786
20	4305	5559	7131	9093	11530
30	43281	53172	65087	79398	96534
40	293111	349468	415674	493289	584105
50	1543219	1802772	2102403	2447799	2845386
60	6804473	7830789	9000327	10331584	11845255
70	26263295	29877560	33954762	38550083	43724846
80	91320685	102932775	115925765	130453251	146684804
90	291792214	326407719	364878976	407609270	455041472
100	869159567	966087237	1073198435	1191501139	1322097384
110	2439632757	2696958618	2979929262	3290959508	3632679048
120	6506678833	7159086457	7873456604	8655363669	9510857088
130	16598077810	18186814659	19919915872	21809834782	23870044778
140	40711717101	44445124546	48504190291	52915934946	57709511909
150	96433642033	104932190253	114144413554	124127420336	134942673867

TABLES OF $A_u(W)$ FOR COMPUTING PROBABILITIES

W	5	6	7	8	9
	23	37	57	85	125
10	866	1146	1503	1956	2525
20	10237	12690	15663	19250	23564
30	73718	88203	105246	125256	148702
40	397059	464345	542024	631567	734620
50	1759194	2024291	2326155	2669470	3059483
60	6760033	7684433	8725976	9898428	11217065
70	23283841	26214912	29489851	33146281	37225630
80	73486831	82094957	91647122	102240249	113980619
90	215869545	239595822	265772791	294638397	326452330
100	597031924	659011740	727057904	801730347	883637667
110	1568317735	1722930349	1891953869	2076658338	2278419743
120	3939821007	4310323209	4713846892	5153181745	5631340947
130	9517040745	10373998574	11304246143	12313737479	13408886008
140	22204650387	24125288176	26204016857	28453220690	30886207570
150	50222667065	54407280722	58924262692	63798780185	69057817615
160	110460359974	119347696861	128917723731	139220513330	150309645406
170	236862430799	255304075033	275118519153	296403476200	319263310774
180	496292226072	533755215811	573925507331	616990415447	663149665123
190	1018043407318	1092675810762	1172552396341	1258026385398	1349473806907

TABLES OF $A_5(W)$ FOR COMPUTING PROBABILITIES

W	5	6	7	8	9
	24	39	61	93	139
10	1069	1437	1914	2528	3311
20	14542	18249	22794	28343	35094
30	116999	141375	170333	204654	245236
40	690170	813813	957698	1124856	1318725
50	3302413	3827063	4428558	5117269	5904869
60	13564506	15515222	17726303	20230012	23062320
70	49547136	56092472	63444613	71696364	80950476
80	164807516	185027732	207572887	232693500	260665423
90	507661759	566003541	630651767	702247667	781493802
100	1466191491	1625098977	1800256339	1993231486	2205735051
110	4007950492	4419888967	4871883131	5367617846	5911098791
120	10446499840	11469409666	12587303496	13808545414	15142198035
130	26115118555	28560813233	31224162015	34123572261	37278930786
140	62916367488	68570412722	74708207148	81369155636	88595719479
150	146656309098	159339470975	173068676246	187926200521	204000491482

TABLES OF $A_6(W)$ FOR COMPUTING PROBABILITIES

W	0	1	2	3	4
	1	2	4	8	14
10	217	315	451	635	883
20	5188	6775	8785	11322	14509
30	57790	71660	88524	108967	133662
40	426773	512883	614839	735282	877283
50	2420502	2847145	3342989	3918380	4585081
60	11389554	13187209	15247892	17607287	20305513
70	46568808	53265315	60859538	69463595	79202750
80	170523435	193149518	218587045	247163179	279240706
90	571032920	641629306	720430143	808331675	906322017
100	1775481584	1981577598	2210234896	2463782212	2744775194
110	5184407951	5752975586	6380509823	7072793445	7836145430

TABLES OF $A_7(W)$ FOR COMPUTING PROBABILITIES

W	0	1	2	3	4
	1	2	4	8	14
10	225	329	475	675	947
20	5863	7722	10100	13127	16963
30	70917	88623	110320	136825	169094
40	563598	681977	823104	990963	1190193
50	3411465	4037338	4769163	5623489	6619260
60	17013043	19806469	23025984	26731963	30992681
70	73300771	84257996	96744889	110960783	127129305

TABLES OF $A_8(W)$ FOR COMPUTING PROBABILITIES

W	0	1	2	3	4
	1	2	4	8	14
10	229	337	489	699	987
20	6352	8471	11087	14506	18868
30	82004	103129	129188	161228	200489

TABLES OF $A_6(W)$ FOR COMPUTING PROBABILITIES

W	5	6	7	8	9
	24	40	63	97	147
10	1216	1654	2229	2979	3946
20	18488	23437	29569	37128	46416
30	163415	199165	241993	293178	354203
40	1044373	1240586	1470581	1739695	2054007
50	5356420	6247565	7275703	8460258	9823249
60	23387755	26904776	30913512	35477904	40669607
70	90216743	102661280	116709928	132555902	150414071
80	315221587	355551167	400722405	451280545	507828602
90	1015490361	1137036461	1272281073	1422677810	1589825477
100	3056016968	3400580561	3781833937	4203466382	4669517263
110	8677467755	9604296918	10624858717	11748127669	12963892236

TABLES OF $SA_7(W)$ FOR COMPUTING PROBABILITIES

W	5	6	7	8	9
	24	40	64	99	151
10	1315	1805	2454	3308	4421
20	71796	27858	35432	44850	56516
30	208265	255681	312910	381801	464523
40	1426174	1705109	2034179	2421672	2877111
50	7778092	9124676	10687168	12497596	14592412
60	35885351	41497188	47926555	55284373	63695591
70	145501116	166356871	190010699	216813898	247158960

TABLES OF $A_8(W)$ FOR COMPUTING PROBABILITIES

W	5	6	7	8	9
	24	40	64	100	153
10	1479	1905	2607	3537	4758
20	24403	31395	40190	51202	64937
30	248450	306883	377847	463805	567652

TABLES OF $SA_1(W)$ FOR COMPUTING PROBABILITIES

W	0	1	2	3	4
10	162	217	287	375	485
20	1929	2376	2912	3552	4314
30	12805	15180	17945	21158	24883
40	62792	72749	84132	97125	111934
50	253872	289552	329814	375196	426296
60	895260	1009319	1136772	1279073	1437818
70	2847063	3181015	3551313	3961649	4416025
80	8344193	9255554	10259779	11365702	12582947
90	22879489	25226450	27798996	30617462	33703896
100	59338017	65093522	71374043	78224642	85694025
110	146765765	160300066	175011805	190997662	208361823
120	348414727	379101425	412342407	448341215	487314423
130	797923896	865288345	938045344	1015604351	1101404332
140	1770102262	1913817631	2068619968	2235322239	2414794112
150	3816629480	4115453304	4436549640	4781499294	5151989930
160	8021098902	8628167847	9279048902	9976757989	10724509132
170	16470136894	17677620635	18969648158	20351874127	21830314646
180	33109746103	35465475373	37981498531	40668249113	43536810439
190	65278991375	69793666178	74607757589	79739633256	85209613553
200	126419407715	134931406912	143992868445	153637803162	163902241175
210	240800887288	256606116622	273406989292	291263714552	310239995300
220	451667552784	480601729270	511316657639	543917647692	578515994801
230	835123433173	887395539150	942813929997	1001560744780	1063828265867
240	1523557106993	1616827014550	1715592519355	1820165325792	1930874160103
250	2744772578515	2909267689335	3083258958363	3267274528281	3461870845463
260	4886755818232	5173701473435	5476890011198	5797207908705	6135588297186
270	8603938482206	9099330635375	9622239842426	10174140821963	10756584560238
280	14990108847582	15837033562267	16730152173984	17671897304795	18664825280278
290	25857566946478	27292094411340	28603501715946	30395769439688	32073077309180

TABLES OF $SA_1(W)$ FOR COMPUTING PROBABILITIES

W	5	6	7	8	9
	28	42	61	86	119
10	622	791	998	1251	1558
20	5218	6287	7548	9031	10770
30	29193	34171	39909	46511	54095
40	128791	147952	169703	194364	222289
50	483774	548360	620863	702174	793277
60	1614763	1811840	2031167	2275070	2546103
70	4918847	5474919	6089490	6768295	7517588
80	13921984	15394205	17012004	18788849	20739376
90	37082204	40778300	44820252	49238460	54065842
100	93834826	102703887	112362588	122877195	134319208
110	227216512	247682589	269890187	293979363	320100829
120	529494717	575132038	624494769	677871044	735570135
130	1192915793	1291642898	1398125779	1512942986	1636714049
140	2607965684	2815831500	3039454818	3279972088	3538597776
150	5549822987	5976921004	6435335335	6927254412	7455012497
160	11525726899	12384059510	13303392825	14287865148	15341882829
170	23411369279	25101844461	26908978259	28840466479	30904490401
180	46598954407	49867182471	53354768850	57075806327	61045254620
190	91039038754	97250539823	103868113314	110917200407	118424770188
200	174824346772	186444539793	198805623600	211952915853	225934410806
210	330403225104	351824693628	374579802862	398748295122	424414493150
220	615229307030	654181849219	695504905394	739337160097	785825099408
230	1129819462280	1199748561381	1273841649940	1352337305823	1435487262412
240	2048065665736	2172105343973	2303378541829	2442291490207	2589272394297
250	3667634121983	3885181870051	4115164513183	4358267077243	4615210964885
260	6493013329813	6870516666060	7269186077443	7690166180018	8134661300577
270	11371202118417	12019708623196	12703907448935	13425694601316	14187063311137
280	19711622205283	20815110327372	21978254702448	23204170175548	24496128690736
290	33839813826612	35700586348912	37660231637150	39723826896844	41896701332836

TABLES OF $SA_2(W)$ FOR COMPUTING PROBABILITIES

W	0	1	2	3	4
	1	3	7	14	25
10	315	439	602	814	1087
20	5058	6368	7970	9920	12284
30	40407	48636	58352	69794	83235
40	226709	265740	310841	362865	422775
50	1018963	1173200	1348777	1548396	1775073
60	3920867	4455629	5057639	5734702	6495457
70	13429430	15107750	16980749	19069399	21396774
80	41984181	46850878	52243960	58216580	64826907
90	121889450	135116394	149688446	165733856	183392342
100	332747119	366795676	404121162	445020318	489815187
110	862162034	945837006	1037173839	1136834668	1245535662
120	2135614341	2333143621	2547956748	2781484836	3035271171
130	5086198149	5536332802	6024243493	6552937153	7125647825
140	11700335971	12694510405	13768955939	14929832644	16183750051
150	26096183119	28231304226	30532732759	33012803520	35684722689
160	56609977007	61081142186	65889045909	71057900175	76613555041
170	119755616593	128906523363	138725264751	149258397490	160555579397
180	247607636440	265948710802	285589134971	306616959915	329125945410
190	501347925583	537409803970	575955683172	617149437226	661165296725
200	995752444670	1065412766394	1139745313115	1219050569556	1303647554290
210	1942845903224	2075221067356	2216252892516	2366484781908	2526492887816
220	3728733429350	3976484032294	4240050086989	4520401679986	4818566081790
230	7047208524279	7504356530968	7990022454276	8505917275748	9053850720143
240	13129493694340	13961892664930	14845086213695	15782057990722	16775960373798
250	24135129786493	25632042281919	27218388744856	28899316810200	30680259590319
260	43810464355845	46471027883686	49287354367043	52268235792391	55422942664229

TABLES OF $SA_3(W)$ FOR COMPUTING PROBABILITIES

W	0	1	2	3	4
	1	3	7	15	28
10	446	641	906	1260	1728
20	9220	11839	15100	19140	24123
30	87216	106476	129519	157010	189711
40	555316	548163	778248	918181	1080938
50	2761260	3208195	3721333	4309656	4983268
60	11557168	13234739	15137038	17291870	19730196
70	42540258	48176620	54506651	61609657	69573394
80	141635311	158985763	178320634	199851891	223812670
90	434852688	484585553	539651393	600586544	667977895
100	1248361326	1382672231	1530580502	1693381644	1872487418
110	3385867184	3730668921	4108611497	4522701852	4976204583
120	8745760291	9593003912	10517919047	11527245127	12628275083
130	21649947554	23653505655	25832936286	28202884707	30779153480
140	51623468176	56203533346	61169824143	66553300820	72387283397
150	119060719662	129218480336	140201466052	152073523182	164903203025
160	266511161874	288434132850	312077187123	337569062049	365047687891
170	580690324383	626852817259	676520844079	729948721873	787408396656
180	1234594778124	1329637211987	1431678516131	1541211738039	1658763157397
190	2566681766152	2758373168631	2963776486884	3183831203378	3419538465356
200	5227347499320	5606710766213	6012473712523	6446398068876	6910358320503
210	10445884763655	11183614710143	11971358006209	12812369545563	13710107597959
220	20510308837183	21921795922081	23426619517020	25030710517169	26740362003871

TABLES OF $SA_2(W)$ FOR COMPUTING PROBABILITIES

W	5	6	7	8	9
	42	67	103	153	222
10	1436	1878	2434	3129	3992
20	15138	18571	22686	27602	33456
30	98987	117406	138896	163917	192991
40	491656	570727	661359	765091	883648
50	2032170	2323433	2653033	3025607	3446310
60	7349465	8307297	9380632	10582367	11926735
70	23988246	26871693	30077736	33639988	37595324
80	72138564	80221112	89150568	99009961	109889944
90	202816060	224170642	247636312	273409102	301702154
100	538855144	592519074	651217732	715396269	785536940
110	1364051180	1493218251	1633941367	1787197614	1954042194
120	3110979553	3610403209	3935474322	4288274253	4671044417
130	7745852946	8417290723	9143978725	9930233709	1078069271
140	17537798328	18999581551	20577253146	22279553639	24115850927
150	38562626507	41661643693	44997961842	48588898105	52452974339
160	82583627074	88997614551	95887019899	103285479699	111228902728
170	172669766769	185657423858	199578745028	214497890337	230483235429
180	353215914322	378993127881	406570683172	436068934208	467615937792
190	708188475980	758415836548	812056589294	869333036955	930481359482
200	1393874916328	1490092094083	1592680539928	1702045013936	1818614950734
210	2696888007012	2878317581444	3071467809874	3277065876566	3495882303024
220	5135630987016	5472747931009	5831135892410	6212085091106	6616960991818
230	9635736738028	10253599281524	10909578387968	11605936587347	12345065650380
240	17830123656458	18948065717771	20133502198287	21390357207978	22722774592584
250	32566950932183	34565441460370	36682115445366	38923708537613	41297326410251
260	58761249122204	62293459330289	66030435199647	69983625510307	74165096500224

TABLES OF $SA_3(W)$ FOR COMPUTING PROBABILITIES

W	5	6	7	8	9
	49	82	131	202	304
10	2342	3138	4162	5471	7130
20	30238	37711	46809	57840	71167
30	228506	274416	328607	392423	467407
40	1269904	1488908	1742297	2034995	2372550
50	5753503	6633089	7636301	8779110	10079399
60	22486503	25599167	29110828	33068870	37525907
70	78494897	88481350	99651130	112134885	126076674
80	250459198	280073003	312963238	349469159	389962947
90	742467453	824757186	915614207	1015876555	1126459340
100	2069435646	2285900718	2523705150	2784831915	3071437658
110	5472662677	6015920103	6610145950	7259860291	7969962290
120	13828898600	15137648336	16563749405	18117172853	19808692794
130	33578789232	36620175430	39923132205	43509022941	47400868204
140	78707622471	85552882371	92964536543	100987176110	109668733293
150	178764092559	193735166875	209901164867	227352990664	246188141214
160	394660814197	426566676600	460934707790	497946293896	537795579328
170	849190610848	915606145731	986987141684	1063688501185	1146089381163
180	1784894430453	1920204865920	2065333840569	2220963364661	238782080371
190	3671964962864	3942247039926	4231595054650	4541297999819	4872728399400
200	7406348628851	7936490162959	8503038860431	9108393642787	975510511259
210	14668246013221	15690687127007	16781575407833	17945311889787	1918656943603
220	28562250504036	30503458448178	32571497896281	34774335595142	3712041943999

TABLES OF $SA_u(W)$ FOR COMPUTING PROBABILITIES

W	0	1	2	3	4
	1	3	7	15	29
10	535	787	1137	1616	2263
20	13495	17617	22834	29399	37618
30	147761	182684	224977	276032	337472
40	1054709	1262802	1508143	1796778	2135647
50	5758311	6747270	7891975	9214918	10741579
60	26082232	30085958	34657727	39872042	45812428
70	102797152	117159490	133387949	151708020	172370546
80	363504610	410344913	462826065	521582952	587317280
90	1177751756	1319131743	1476437832	1651353602	1845729651
100	3549556344	3949640040	4392186708	4881440180	5422043762
110	10063954610	11135814251	12315487174	13613215196	15040159193
120	27077167568	29816365058	32817938531	36105684354	39705442651
130	69605534421	76322939414	83655551790	91656841158	100384687901
140	171899195396	187783506709	205063534985	223855897670	244286478793
150	409677137018	446038914549	485475120955	528233385655	574580340043
160	945721449704	1026568783372	1114010517830	1208560210371	1310769137595
170	2121267518813	2296347735420	2485236275570	2688973798770	2908675915469
180	4635437199425	5005598152441	5404042932785	5832839465467	6294200356822
190	9890929564857	10656546474667	11478940208367	12362144589984	13310468030213

TABLES OF $SA_s(W)$ FOR COMPUTING PROBABILITIES

W	0	1	2	3	4
	1	3	7	15	29
10	588	879	1290	1862	2648
20	17212	22771	29902	38995	50525
30	212828	266000	331087	410485	507019
40	1678727	2028195	2443869	2937158	3521263
50	9969744	11772516	13874919	16322718	19168104
60	48552749	56383538	65383865	75715449	87560704
70	203922362	233799922	267754684	306304767	350029613
80	763081359	866014134	981939899	1112393150	1259077954
90	2601637226	2928044945	3292973921	3700533191	4155574663
100	8212792766	9178880003	10252078438	11443579577	12765676961
110	24295823062	26992781680	29972710942	33263670450	36896349498
120	67981467558	75140554015	83014010619	91669374288	101180231376
130	181232265637	199419080296	219338996168	241148830950	265018875728
140	463033189679	507478314225	555982504516	608998439462	666607951371
150	1139201455877	1244133646130	1358278059684	1482405480020	1617348153887

TABLES OF $SA_4(W)$ FOR COMPUTING PROBABILITIES

W	5	6	7	8	9
	52	89	146	231	356
10	3129	4275	5778	7734	10259
20	47855	60545	76208	95458	119022
30	411190	499393	604639	729895	878597
40	2532706	2997051	3539075	4170642	4905262
50	12500773	14525064	16851219	19520689	22580172
60	52572461	60256894	68982870	78881298	90098363
70	195654387	221869299	251359150	284505431	321731061
80	660804111	742899068	834546190	936786439	1050767058
90	2061599196	2301195018	2566467809	2861606206	3188058536
100	6019075686	6678087426	7405145330	8206875677	9090513344
110	16608476928	1833147277	20223361146	22300019484	24578439227
120	43645263658	47955586867	52669433759	57822615504	63453956451
130	109901728646	120275727220	131579973363	143893710842	157302596850
140	266491129180	290616417356	316820434213	345273654903	376159862473
150	624803007108	679210287830	738134550522	801933330707	870991148322
160	1421229497569	1540577194430	1669494918161	1808715431491	1959025076897
170	3145538346268	3400842421301	3675960940454	3972364416654	4291627727428
180	6790492582894	7324247798705	7898173306036	8515163721483	9178313386606
190	14328511437531	15421187248293	16593739644634	17851766030032	19201239836939

TABLES OF $SA_5(W)$ FOR COMPUTING PROBABILITIES

W	5	6	7	8	9
	53	92	153	246	385
10	3717	5154	7068	9596	12907
20	65067	83316	106110	134453	169547
30	624018	765393	935726	1140380	1385616
40	4211433	5025246	5982944	7107800	8426525
50	22470517	26297580	30726138	35843407	41748276
60	101125210	116640432	134366735	154596747	177659067
70	399576749	455669221	519113834	590810198	671760674
80	1423885470	1608913202	1816486089	2049179589	2309845012
90	4663236422	5229239963	5859891730	6562139397	7343633199
100	14231868452	15856967429	17657223768	19650455254	21856190305
110	40904299990	45324188957	50196072088	55563689934	61474788725
120	111626731216	123096140882	135683444378	149491989792	164634187827
130	291133994283	319694807516	350918969531	385042541792	422321472578
140	729524318859	798094731581	872802938729	954172094365	1042767813844
150	1764004462985	1923343933960	2096412610206	2284338810727	2488339302209

TABLES OF $SA_6(W)$ FOR COMPUTING PROBABILITIES

W	0	1	2	3	4
	1	3	7	15	29
10	617	932	1383	2018	2901
20	20114	26888	35673	46995	61504
30	274332	345992	434516	543483	677145
40	2355872	2868755	3483594	4218876	5096159
50	15065903	17913048	21256037	25174417	29759498
60	78312247	91499456	106747348	124354635	144660148
70	348582510	401847825	462707363	532170958	611373708
80	1374455067	1567604585	1786191630	2033354809	2312595515
90	4914232741	5555862047	6276292190	7084623865	7990945882
100	16203738648	18185316246	20395551142	22859333354	25604108548
110	49899931610	55652907196	62033417019	69106210484	76942355894

TABLES OF $SA_7(W)$ FOR COMPUTING PROBABILITIES

	1	2	3	4	
	1	3	7	15	29
10	632	961	1436	2111	3058
20	22224	29946	40046	53173	70136
30	327505	416128	526448	663273	832367
40	3019145	3701122	4524226	5515189	6705382
50	20581092	24618430	29387593	35011082	41630342
60	113323329	133129798	156155782	182887745	213880426
70	531470255	615728251	712473140	823433923	950563228

TABLES OF $SA_8(W)$ FOR COMPUTING PROBABILITIES

W	0	1	2	3	4
	1	3	7	15	29
10	639	976	1465	2164	3151
20	23689	32110	43197	57703	76571
30	370702	473831	603019	764247	964736

TABLES OF $SA_6(W)$ FOR COMPUTING PROBABILITIES

W	5	6	7	8	9
	53	93	156	253	400
10	4117	5771	8000	10979	14925
20	79992	103429	132998	170126	216542
30	840560	1039725	1281718	1574896	1929099
40	6140532	7381118	8851699	10591394	12645401
50	35115918	41363483	48639186	57099444	66922693
60	168047903	194952679	225866191	261344095	302013702
70	701590451	804251731	920961659	1053517561	1203931632
80	2627817102	2983368269	3384090674	3835371219	4343199821
90	9006436243	10143472704	11415753777	12838431587	14428257064
100	28660125516	32060706077	35842540014	40046006396	44715523659
110	85619823649	95224120567	105848979284	117597106953	130580999189

TABLES OF $SA_7(W)$ FOR COMPUTING PROBABILITIES

W	5	6	7	8	9
	53	93	156	256	407
10	4373	6178	8632	11940	16361
20	91932	119790	155222	200072	256588
30	1040632	1296313	1609223	1991024	2455547
40	8131556	9836665	11870844	14292516	17169627
50	49408434	58533110	69220278	81717874	96310286
60	249765777	291262965	339189520	394473893	458169484
70	1096064344	1262421215	1452431914	1669245812	1916404772

TABLES OF $SA_8(W)$ FOR COMPUTING PROBABILITIES

W	5	6	7	8	9
	53	93	157	257	410
10	4530	6435	9042	12579	17337
20	100974	132369	172559	223761	288698
30	1213191	1520074	1897921	2361726	2929378